Brilliant

SOLVED Question Paper 2022

MATHEMATICS

Our Useful Books for BSEB, CBSE, JAC & All Other Competitive Exams

for CLASS 12

Our Highly Useful Question Bank

for CLASS 12

Total No. of Questions : 138] **Solved Question Paper : 2022** [Marks : 100

Time : 3 Hr. 15 Min.]

MATHEMATICS

Section–A
(Objective Type Questions)

Question Nos. 1 to 100 have four options, out of which only one is correct. Answer any 50 questions. You have to mark your selected option on the OMR-Sheet. 50 × 1 = 50

1. $\frac{d}{dx}\left(e^{13x}\right) =$

(A) e^{13x} (B) $\frac{1}{13}e^{13x}$ (C) $13e^{13x}$ (D) $-13e^{13x}$ Ans.—(C)

2. $\frac{d}{dx}(\sin\frac{x}{5}) =$

(A) $\cos\frac{x}{5}$ (B) $-\frac{1}{5}\cos\frac{x}{5}$

(C) $\frac{1}{5}\cos\frac{x}{5}$ (D) $-\frac{1}{5}\sin\frac{x}{5}$ Ans.—(C)

3. $\frac{d}{dx}\cos 2x =$

(A) $-\sin 2x$ (B) $2\sin 2x$

(C) $-2\sin 2x$ (D) $-\frac{1}{2}\sin 2x$ Ans.—(C)

4. $\frac{d}{dx}(\tan\frac{x}{3}) =$

(A) $\sec^2\frac{x}{3}$ (B) $\frac{1}{3}\sec^2\frac{x}{3}$

(C) $3\sec^2\frac{x}{3}$ (D) $3.\cot\frac{x}{3}$ Ans.—(B)

5. $\frac{d}{dx}\left(\frac{x^4}{4}\right) =$

(A) $4x^3$ (B) $\frac{x^3}{4}$ (C) x^3 (D) $16x^3$ Ans.—(C)

6. $\frac{d}{dx}(\log x^4) =$

(A) $\frac{4}{x}$ (B) $\frac{1}{x^4}$ (C) $\frac{1}{4x}$ (D) $\frac{3}{4x}$ Ans.—(A)

7. $\frac{d}{d\theta}(\sin^2\theta) =$

(A) $\sin^2\theta$ (B) $\cos^2\theta$ (C) $\sin 2\theta$ (D) $-\sin 2\theta$ Ans.—(C)

8. $\frac{d}{d\theta}(\cos^3\theta) =$

(A) $-3\sin^3\theta$ (B) $3\sin^3\theta\cos\theta$

(C) $-3\cos^2\theta\sin\theta$ (D) $3\cos^2\theta\sin\theta$ Ans.—(C)

9. $\frac{d}{dx}(\tan^2 x) =$

(A) $\sec^2 x$ (B) $2\tan x$

(C) $-2\tan x\sec^2 x$ (D) $2\tan x\sec^2 x$ Ans.—(D)

10. $\int x^{3/2}dx =$

(A) $\frac{2}{3}x^{\frac{3}{2}}+k$ (B) $\frac{2}{5}x^{\frac{5}{2}}+k$

(C) $\frac{2}{3}x^{\frac{5}{2}}+k$ (D) $\frac{5}{2}x^{\frac{5}{2}}+k$ Ans.—(B)

11. $\int\frac{dx}{x} =$

(A) $\log|x|+k$ (B) $x\log|x|+k$

(C) e^x+k (D) $x+k$ Ans.—(A)

12. $\int\sin x\,dx =$

(A) $k+\cos x$ (B) $k-\cos x$

(C) $-\tan x+k$ (D) $\tan x-k$ Ans.—(B)

13. $\int\cos 6\theta\;d\theta =$

(A) $k+\sin 6\theta$ (B) $k-\sin 6\theta$

(C) $k-6\sin 6\theta$ (D) $k+\frac{1}{6}\sin 6\theta$ Ans.—(D)

14. $\int\tan 2x\;dx =$

(A) $\frac{1}{2}\log|\tan 2x|+k$ (B) $\frac{1}{2}\log|\sec 2x|+k$

(C) $2\log|\sec 2x|+k$ (D) $\log|\sec x|+k$ Ans.—(B)

15. $\int_0^1 x^2dx =$

(A) $\frac{1}{3}$ (B) 0 (C) 1 (D) $-\frac{1}{3}$ Ans.—(A)

16. $\int_0^{\pi/2}\sin x\,dx =$

(A) 0 (B) 1 (C) -1 (D) 2 Ans.—(B)

17. $\int_a^b e^x dx =$

(A) $\frac{b}{a}e^x$ (B) e^{b-a} (C) e^b-e^a (D) e^b+e^a Ans.—(C)

18. $\int_a^b dx =$

(A) $b-a$ (B) $b+a$ (C) $\frac{b}{a}$ (D) ab Ans.—(A)

19. $\int_0^1 x\,dx =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{4}$ Ans.—(B)

20. $\int\frac{x^2-1}{x-1}dx =$

(A) $x+k$ (B) $\frac{x^2}{2}+x+k$

(C) $\frac{x^3}{3}-\frac{x^2}{2}+k$ (D) $\frac{x^2}{2}+2x+k$ Ans.—(B)

21. $\int_{-\pi}^{\pi}\tan x\,dx =$

(A) 0 (B) 1 (C) 2 (D) -1 Ans.—(A)

22. $\int_{-\pi/4}^{\pi/4}\sin^3 x\,dx =$

(A) 0 (B) 1 (C) 2 (D) $\frac{3}{\sqrt{2}}$ Ans.—(A)

23. $\int\frac{dx}{e^x+e^{-x}} =$

(A) $\tan^{-1}(e^x)+k$ (B) $\tan^{-1}(e^{-x})+k$

(C) $\log|e^x+e^{-x}|+k$ (D) $x+k$ Ans.—(A)

24. $\int (4e^{3x}+1)\,dx =$

(A) $4e^{3x} + k$ (B) $\frac{4}{3}e^{3x} + x + k$

(C) $12e^{3x} + x + k$ (D) $12e^{3x} + k$ Ans.—(B)

25. $3\int \sqrt{x}\,dx =$

(A) $2x^{3/2} + k$ (B) $4x^3 + k$

(C) $3x^{3/2} + k$ (D) $6x^{3/2} + k$ Ans.—(A)

26. $\int \frac{dx}{\sqrt{1-x^2}} =$

(A) $\tan^{-1} x + k$ (B) $\sin^{-1} x + k$

(C) $\cos^{-1} x + k$ (D) $\sec^{-1} x + k$ Ans.—(B)

27. $2\int \frac{1}{1+4x^2}dx =$

(A) $\tan^{-1} x + k$ (B) $\log|1+4x^2| + k$

(C) $\tan^{-1} 2x + k$ (D) $\sin^{-1} 2x + k$ Ans.—(C)

28. $\int \left(\frac{\cos^2 x + \sin^2 x}{\operatorname{cosec} x}\right) dx =$

(A) $k + \operatorname{cosec} x$ (B) $k - \sin x$

(C) $k + \sin x$ (D) $k - \cos x$ Ans.—(D)

29. $3\int dx =$

(A) $3x + k$ (B) $3 + k$

(C) $\frac{1}{3} + k$ (D) $1 + k$ Ans.—(A)

30. $\frac{d}{dx}(a^x) =$

(A) a^x (B) $\frac{a^x}{\log a}$ (C) $a^x.\log a$ (D) $x \log a$ Ans.—(C)

31. The solution of the differential equation $x\,dy + y\,dx = 0$ is

(A) $x = y + c$ (B) $xy = c$

(C) $x + y = c$ (D) $x = y^2 + c$ Ans.—(B)

32. $\int \frac{dx}{x^2-1} =$

(A) $\sin^{-1} x + k$ (B) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + k$

(C) $\frac{1}{2}\log\left|\frac{x+1}{x-1}\right| + k$ (D) $\sqrt{1-x^2} + k$ Ans.—(B)

33. $\int \frac{dx}{x^2+4} =$

(A) $\frac{1}{2}\sin^{-1}\frac{x}{2} + k$ (B) $\frac{1}{2}\cos^{-1}\frac{x}{2} + k$

(C) $\frac{1}{2}\tan^{-1}\frac{x}{2} + k$ (D) $\tan^{-1} x + k$ Ans.—(C)

34. $2\vec{i}.3\vec{j} =$

(A) $6\vec{k}$ (B) 6 (C) 0 (D) 1 Ans.—(C)

35. $3\vec{j}.2\vec{k} =$

(A) 6 (B) 1 (C) $6\vec{i}$ (D) 0 Ans.—(D)

36. $(3\vec{i}+4\vec{j}-5\vec{k}).\vec{i} =$

(A) 7 (B) 3 (C) 2 (D) 0 Ans.—(B)

37. $(\vec{i}+\vec{j}+\vec{k}).(2\vec{i}+3\vec{j}) =$

(A) 5 (B) 6 (C) 1 (D) 11 Ans.—(A)

38. $|2\vec{i}+\vec{j}+2\vec{k}| =$

(A) 3 (B) 5 (C) 2 (D) 1 Ans.—(A)

39. $\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx =$

(A) $\frac{\pi^2}{8}$ (B) $\frac{\pi^2}{12}$ (C) $\frac{\pi^2}{16}$ (D) $\frac{\pi^2}{32}$ Ans.—(D)

40. If $\vec{a} = \vec{i}+2\vec{j}$ and $\vec{b} = 2\vec{i}+\vec{j}$ then the value of $|\vec{a}+\vec{b}|$ is

(A) $\sqrt{2}$ (B) $2\sqrt{2}$ (C) $3\sqrt{2}$ (D) $4\sqrt{3}$ Ans.—(C)

41. The unit vector in the direction of the vector $\vec{i}+\vec{j}$ is

(A) $\frac{\vec{i}+\vec{j}}{\sqrt{2}}$ (B) $2\vec{i}+3\vec{j}$

(C) $\vec{i}-\vec{j}+\vec{k}$ (D) $2\vec{i}-3\vec{j}$ Ans.—(A)

42. The solution of the differential equation $\frac{dy}{dx}+y=1,\ y \neq 1$ is

(A) $y = 1 + Ae^{-x}$ (B) $y = Ae^{x}$

(C) $y = Ae^{2x} + 1$ (D) $y = 1 + Ae^{3x}$ Ans.—(A)

43. The integrating factor of the differential eqation $\frac{dy}{dx}+2y=\cos x$ is

(A) e^{2x} (B) e^{-2x}

(C) $e^{\cos x}$ (D) none of these Ans.—(A)

44. $5\vec{j}\times 4\vec{i} =$

(A) 20 (B) -20 (C) $20\vec{k}$ (D) $-20\vec{k}$ Ans.—(D)

45. The distance of the plane $2x - 3y + 4z = 6$ from the origin is

(A) $\frac{6}{\sqrt{35}}$ (B) $\frac{6}{\sqrt{37}}$ (C) $\frac{6}{\sqrt{29}}$ (D) $\frac{6}{\sqrt{31}}$ Ans.—(C)

46. The angle between two planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{4}{21}\right)$ (D) $\cos^{-1}\left(\frac{16}{21}\right)$ Ans.—(C)

47. The direction cosines of a line having direction ratios 2, −1, −2 are

(A) $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ (B) $\frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$

(C) $\frac{2}{5}, -\frac{1}{5}, \frac{-2}{5}$ (D) none of these Ans.—(A)

48. $(\vec{i}-2\vec{j}+5\vec{k}).(-2\vec{i}+4\vec{j}+2\vec{k}) =$

(A) 0 (B) 2 (C) −4 (D) 6 Ans.—(A)

49. $\int xe^{2x}dx =$

(A) $(2x-1)\frac{e^{2x}}{4} + k$ (B) $(x-1)\frac{e^{2x}}{4} + k$

(C) $\frac{xe^{2x}}{4} + k$ (D) $\frac{(x+2)e^{2x}}{4} + k$ Ans.—(A)

50. $(2\vec{i}-3\vec{j}+5\vec{k}).(2\vec{i}+2\vec{j}+2\vec{k}) =$

(A) 8 (B) 2 (C) 4 (D) 20 Ans.—(A)

51. If the direction cosines of a straight line are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, x$ then the value of x is
(A) $\frac{6}{\sqrt{77}}$ (B) $\frac{8}{\sqrt{77}}$ (C) $\frac{9}{\sqrt{77}}$ (D) $\frac{1}{\sqrt{77}}$ Ans.—(B)

52. $(\vec{i}+\vec{j}+\vec{k}).(\vec{i}-\vec{j}+\vec{k})\times(\vec{i}+2\vec{j}-\vec{k})=$
(A) 0 (B) 2 (C) 4 (D) 6 Ans.—(C)

53. The direction ratios of the normal to the plane 3x + 4y + 5z – 6 = 0 are
(A) 3, 4, 5 (B) –3, 4, 5
(C) 3, –4, 5 (D) 2, 3, –4 Ans.—(A)

54. Which of the following planes is parallel to the plane $x = 0$?
(A) x = –5 (B) y = 0
(C) z = 5 (D) none of these Ans.—(A)

55. Equation of a plane parallel to the plane 2x – 3y + 4z = 7 is
(A) 2x – 3y – 4z = 7 (B) 2x – 3y + 4z = 11
(C) 2x + 4y – 3z = 11 (D) none of these Ans.—(B)

56. Equation of a plane perpendicular to the plane 4x + 3y – z + 1 = 0 is
(A) x – 5y – 11z + 7 = 0 (B) x – y – z = 2
(C) 3x – 11y + 9z = 1 (D) none of these Ans.—(A)

57. $\begin{vmatrix} 2 & 5 & 7 \\ 6 & -8 & -2 \\ 3 & 5 & 8 \end{vmatrix} =$
(A) 0 (B) 1 (C) –13 (D) 23 Ans.—(A)

58. $\begin{vmatrix} 4 & 1 & 3 \\ 0 & 2 & -2 \\ 5 & 7 & -2 \end{vmatrix} =$
(A) 0 (B) 1 (C) –1 (D) 19 Ans.—(A)

59. $3\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$
(A) $\begin{bmatrix} 15 & 18 \\ 7 & 8 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 6 \\ 21 & 24 \end{bmatrix}$
(C) $\begin{bmatrix} 15 & 18 \\ 21 & 24 \end{bmatrix}$ (D) $\begin{bmatrix} 15 & 6 \\ 21 & 8 \end{bmatrix}$ Ans.—(C)

60. $2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
(A) $\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 4 \\ 6 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 2 \\ 6 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ Ans.—(D)

61. $\begin{bmatrix} 2 & 5 \\ 8 & 10 \end{bmatrix}\begin{bmatrix} 5 \\ 50 \end{bmatrix} =$
(A) $\begin{bmatrix} 260 & 540 \end{bmatrix}$ (B) $\begin{bmatrix} 260 \\ 540 \end{bmatrix}$
(C) $\begin{bmatrix} 10 & 25 \\ 400 & 500 \end{bmatrix}$ (D) $\begin{bmatrix} 35 \\ 900 \end{bmatrix}$ Ans.—(B)

62. $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} =$
(A) 12 (B) 24 (C) 28 (D) –28 Ans.—(D)

63. If $A=\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ then adjoint A =
(A) $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$
(C) $\begin{bmatrix} 2 & 3 \\ -1 & -4 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$ Ans.—(A)

64. If $x = a \cos \theta$, $y = a \sin \theta$ then $\frac{dy}{dx}$ is equal to
(A) tan θ (B) – cot θ
(C) – tan θ (D) $\sec^2 \theta$ Ans.—(B)

65. If $x = at^2$, $y = 2at$ then $\frac{dy}{dx}$ is equal to
(A) t (B) $\frac{1}{t}$ (C) t^2 (D) none of these
Ans.—(B)

66. $\frac{d^2}{dx^2}(x^2+3x+2) =$
(A) 4 (B) 4x (C) 2x + 3 (D) 2 Ans.—(D)

67. $\frac{d}{dx}\left[\cos^{-1}(\sin x)\right] =$
(A) –1 (B) $\frac{x}{\sqrt{1-x^2}}$
(C) $\frac{\sin x}{\sqrt{1-x^2}}$ (D) $\frac{\pi}{2}-x$ Ans.—(A)

68. Which of the following is an objective function ?
(A) $z = 5x + 7y$ (B) $x > 0$
(C) $y > 0$ (D) none of these Ans.—(A)

69. The minimum value of 11x + 2y subject to constraints $x + y \le 7$, $x \ge 0$, $y \ge 0$ is
(A) 77 (B) 14 (C) 0 (D) –14 Ans.—(C)

70. The maximum value of x + y subject to constraints $3x + 5y \le 30$, $x \ge 0$, $y \ge 0$ is
(A) 16 (B) 10 (C) 6 (D) none of these
Ans.—(B)

71. $X=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow X^8 =$
(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 8 \\ 8 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Ans.—(D)

72. $x \in R$, $\tan^{-1}x + \cot^{-1}x =$
(A) π (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$ Ans.—(B)

73. The principal value of $\text{cosec}^{-1}\,2$ is
(A) $\frac{-\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) none of these
Ans.—(C)

74. Which of the following is true in a binary operation 'o' defined on N by $a\ ob = a^3 + b^3$?
(A) Operation is both associative and commutative
(B) Operation is commutative but not associative
(C) Operation is associative but not commutative
(D) None of these Ans.—(B)

75. How many distinct relations can be defined on the set $A = \{a, b, c\}$?
(A) 2^9 (B) 2^3 (C) 9 (D) none of these
Ans.—(A)

76. $\int_1^4 \sqrt{x}\ dx =$
(A) $\frac{14}{3}$ (B) $\frac{11}{3}$ (C) $\frac{19}{3}$ (D) $\frac{22}{3}$ Ans.—(A)

77. If the plane $ax + by + cz + d = 0$ is parallel to the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-3}{6}$ then
(A) 2a + 3b + 5c = 0 (B) 3a + 4b + 5c = 0
(C) 3a + 4b + 6c = 0 (D) none of these Ans.—(C)

78. If the line $\frac{x-2}{a}=\frac{y-3}{b}=\frac{z-4}{c}$ is parallel to the line $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ then
(A) $4a + 2b + 3c = 0$ (B) $4a = 2b = 3c$
(C) $\frac{a}{4}=\frac{b}{2}=\frac{c}{3}$ (D) none of these **Ans.—(C)**

79. $(2\vec{i}+3\vec{k}).(\vec{i}+\vec{j}+4\vec{k})\times(3\vec{i}+\vec{j}+7\vec{k})=$
(A) 0 (B) 112 (C) 126 (D) 192 **Ans.—(A)**

80. $\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{13}=$
(A) $\tan^{-1}\frac{1}{9}$ (B) $\tan^{-1}\frac{2}{9}$
(C) $\tan^{-1}\frac{20}{91}$ (D) $\tan^{-1}\frac{2}{91}$ **Ans.—(B)**

81. $2\tan^{-1}\frac{1}{2}$
(A) $\tan^{-1}\frac{3}{4}$ (B) $\tan^{-1}\frac{4}{3}$
(C) $\tan^{-1}\frac{7}{12}$ (D) $\tan^{-1}\frac{12}{7}$ **Ans.—(B)**

82. Two events A and B are independent if
(A) $P(A \cup B) = P(A).P(B)$ (B) $P(A\cup B)=\frac{P(A)}{P(B)}$
(C) $P(A\cap B) = P(A).P(B)$ (D) $P(A\cap B)=\frac{P(A)}{P(B)}$ **Ans.—(C)**

83. If $P(A)=\frac{1}{2}, P(B)=0$ then $P(A/B)=$
(A) 0 (B) $\frac{1}{2}$
(C) not defined (D) none of these **Ans.—(C)**

84. $\begin{bmatrix}7 & 6\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=$
(A) $\begin{bmatrix}8 & 6\\ 0 & 0\end{bmatrix}$ (B) $\begin{bmatrix}7 & 0\\ 0 & -1\end{bmatrix}$
(C) $\begin{bmatrix}7 & 6\\ 0 & -1\end{bmatrix}$ (D) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ **Ans.—(C)**

85. If events A and B are independent and $P(A)=\frac{1}{2}, P(A\cup B)=\frac{3}{5}$ and $P(B) = p$ then the value of p is
(A) 5 (B) $\frac{1}{5}$ (C) $\frac{1}{13}$ (D) none of these
Ans.—(B)

86. $\begin{bmatrix}1\\ 0\end{bmatrix}\begin{bmatrix}5 & 6\end{bmatrix}=$
(A) $\begin{bmatrix}5 & 6\end{bmatrix}$ (B) $\begin{bmatrix}5\\ 6\end{bmatrix}$ (C) $\begin{bmatrix}0 & 0\\ 5 & 6\end{bmatrix}$ (D) $\begin{bmatrix}5 & 6\\ 0 & 0\end{bmatrix}$ **Ans.—(D)**

87. $[x\ y] = [2x-1\ \ 7] \Rightarrow$
(A) $x = 3, y = 7$ (B) $x = 1, y = 7$
(C) $x = 0, y = 7$ (D) $x = 3, y = 4$ **Ans.—(B)**

88. If a square matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$ then A^{-1} is equal to
(A) $3A^2 + 2A + 5I$ (B) $-3A^2 - 2A - 5I$
(C) $-3A^2 + 2A - 5I$ (D) none of these **Ans.—(B)**

89. $\int_0^1 x(1-x)^{10}dx=$
(A) $\frac{1}{132}$ (B) $\frac{5}{132}$ (C) $\frac{7}{132}$ (D) $\frac{45}{244}$ **Ans.—(A)**

90. If $x=\frac{1}{5}$ then the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is
(A) $\frac{1}{5}$ (B) $-\frac{1}{5}$ (C) $\frac{\sqrt{24}}{5}$ (D) none of these
Ans.—(B)

91. Which of the following matrices is symmetric ?
(A) $\begin{bmatrix}-1 & 2 & -2\\ -2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0 & 2\\ 0 & 1 & 0\\ 0 & 0 & 5\end{bmatrix}$
(C) $\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$ (D) none of these **Ans.—(D)**

92. If A and B are two events such that $P(A) \neq 0$ and $P(B/A) = 1$ then
(A) $A \subset B$ (B) $B \subset A$
(C) $B = \phi$ (D) $A = \phi$ **Ans.—(A)**

93. $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{12}{13}=$
(A) $\sin^{-1}\frac{36}{65}$ (B) $\sin^{-1}\frac{63}{65}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$ **Ans.—(B)**

94. $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x dx=$
(A) 0 (B) 1 (C) 5 (D) 11 **Ans.—(A)**

95. A coin is tossed 10 times. The probability of getting exactly six heads is
(A) ${}^{10}C_6\left(\frac{1}{2}\right)^6$ (B) ${}^{10}C_6\left(\frac{1}{2}\right)^7$
(C) ${}^{10}C_6\left(\frac{1}{2}\right)^8$ (D) ${}^{10}C_6\left(\frac{1}{2}\right)^{10}$ **Ans.—(D)**

96. The sum of direction cosines of the x–axis is
(A) 1 (B) 2 (C) 3 (D) 4 **Ans.—(A)**

97. The intercepts cut off by the plane $2x + y - z = 5$ on the x-, y- and z-axes respectively are
(A) $\frac{2}{5},\frac{1}{5},\frac{-1}{5}$ (B) $\frac{5}{2},5,5$
(C) $\frac{5}{2},5,-5$ (D) 2, 1, –1 **Ans.—(C)**

98. The minimum value of $z = 2x - 3y$ subject to constraints $x + y \le 5$, $x \ge 0$, $y \ge 0$ is
(A) 0 (B) 10 (C) –15 (D) 20 **Ans.—(C)**

99. The distance of the plane $2x - 3y + 6z + 7 = 0$ from the point $(2, -3, -1)$ is
(A) 2 (B) 3 (C) 6 (D) 7 **Ans.—(A)**

100. The order of the differential equation
$$\left(\frac{d^2y}{dx^2}\right)^3+2\left(\frac{dy}{dx}\right)^3+9y=\sin x \text{ is}$$
(A) 3 (B) 4 (C) 2 (D) none of these
Ans.—(C)

Section–B
Short Answer Type Questions

Question Nos. 1 to 30 are Short Answer Type. Answer any 15 questions. Each question carries 2 marks. $15 \times 2 = 30$

1. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

Solⁿ.— $\underset{A}{(-2, 4, -5)}$ ———— $\underset{B}{(1,2,3)}$

D.r.'s of AB are $1 + 2, 2 - 4, 3 + 5$ i.e. $3, -2, 8$

$\therefore$ D.c.'s of AB are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

2. Integrate : $\int \sqrt{1+\cos 2x}\, dx$.

Solⁿ.— $\int \sqrt{1+\cos 2x}\, dx. = \int \sqrt{2\cos^2 x}\, dx$

$= \sqrt{2}\int \cos x dx = \sqrt{2}\sin x + C$

3. Integrate : $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx$

Solⁿ.— $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx = \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} dx = \int dx = x + C$

4. Prove that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

Solⁿ.—L.H.S. $= 2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2} + \tan^{-1}\left(\frac{1}{7}\right)$ $\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right) = \tan^{-1}\left(\frac{21+4}{28-3}\right)$

$= \tan^{-1}\left(\frac{25}{25}\right) = \tan^{-1}(1) = \tan\frac{\pi}{4} = $ R.H.S.

5. If $A = \begin{bmatrix} a & b \\ x & y \\ x^2 & y^2 \end{bmatrix}$ then find AA' and $A'A$.

Solⁿ.—Given that $A = \begin{bmatrix} a & b \\ x & y \\ x^2 & y^2 \end{bmatrix}$ then $A' = \begin{bmatrix} a & x & x^2 \\ b & y & y^2 \end{bmatrix}$

Now, $AA' = \begin{bmatrix} a & b \\ x & y \\ x^2 & y^2 \end{bmatrix}\begin{bmatrix} a & x & x^2 \\ b & y & y^2 \end{bmatrix} = \begin{bmatrix} a^2+b^2 & ax+by & ax^2+by^2 \\ ax+by & x^2+y^2 & x^3+y^3 \\ ax^2+by^2 & x^3+y^3 & x^4+y^4 \end{bmatrix}$

$A'A = \begin{bmatrix} a & x & x^2 \\ b & y & y^2 \end{bmatrix}\begin{bmatrix} a & b \\ x & y \\ x^2 & y^2 \end{bmatrix} = \begin{bmatrix} a^2+x^2+x^4 & ab+xy+x^2y^2 \\ ab+xy+x^2y^2 & b^2+y^2+y^4 \end{bmatrix}$

6. Find the value of the determinant $\begin{bmatrix} 23 & 12 & 11 \\ 36 & 10 & 26 \\ 63 & 26 & 37 \end{bmatrix}$.

Solⁿ.—Given that $\begin{vmatrix} 23 & 12 & 11 \\ 36 & 10 & 26 \\ 63 & 26 & 37 \end{vmatrix}$

$C_2 \to C_2 + C_3$

$= \begin{vmatrix} 23 & 23 & 11 \\ 36 & 36 & 26 \\ 63 & 63 & 37 \end{vmatrix} = 0$ $[\because C_1$ and C_2 are identical$]$

7. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then find $\frac{dy}{dx}$.

Solⁿ.—Given that $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow$ $y = 2\tan^{-1}x$

then $\frac{dy}{dx} = 2\frac{d}{dx}(\tan^{-1}x) = \frac{2}{1+x^2}$

8. If $y = \log\left(x^2\sqrt{x^2+1}\right)$ then find $\frac{dy}{dx}$.

Solⁿ.—Given that $y = \log\left(x^2\sqrt{x^2+1}\right)$

$\Rightarrow$ $y = \log x^2 + \log\left(x^2+1\right)^{\frac{1}{2}}$

$\Rightarrow$ $y = 2\log x + \frac{1}{2}\log\left(x^2+1\right)$

then $\frac{dy}{dx} = \frac{2}{x} + \frac{1}{2}\frac{1\times(2x)}{\left(x^2+1\right)}$

$= \frac{2}{x} + \frac{x}{x^2+1} = \frac{2x^2+2+x^2}{x\left(x^2+1\right)} = \frac{3x^2+2}{x\left(x^2+1\right)}$

9. If $x = a\cos t + b\sin t$, $y = a\sin t + b\cos t$ then find $\frac{dy}{dx}$.

Solⁿ.—Given that $x = a\cos t + b\sin t$

$\Rightarrow$ $\frac{dx}{dt} = -a\sin t + b\cos t$ and $y = a\sin t + b\cos t$

$\Rightarrow$ $\frac{dy}{dx} = a\cos t - b\sin t$

We know that $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t - b\sin t}{-a\sin t + b\cos t}$

10. If $y = (\sin x)^{\log x}$ then find $\frac{dy}{dx}$

Solⁿ.—Given that $y = (\sin x)^{\log x}$

Taking log both sides, we get

$\log y = \log x \cdot \log \sin x$

Now diff. both sides w.r.t. x

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x}\cdot\log\sin x + \log x\cdot\frac{1}{\sin x}\times\cos x$

$\Rightarrow$ $\frac{dy}{dx} = y\left[\frac{\log\sin x}{x} + \cot x\cdot\log x\right]$

$\Rightarrow$ $\frac{dy}{dx} = (\sin x)^{\log x}\left[\frac{\log\sin x}{x} + \cot x\cdot\log x\right]$

11. Integrate : $\int \frac{x-2}{x^2-3x+2}dx.$

Sol^n.—Let $I = \int \frac{x-2}{x^2-3x+2}dx$

$$I = \int \frac{x-2}{(x-1)(x-2)}dx \quad = \int \frac{dx}{x-1} \quad = \log|x-1| + C$$

12. Integrate : $\int \frac{(\sin x - \cos x)^3}{\sqrt{1-\sin 2x}}dx.$

Sol^n.—$\int \frac{(\sin x-\cos x)^3}{\sqrt{1-\sin 2x}}dx = \int \frac{(\sin x-\cos x)^3}{\sqrt{(\sin x-\cos x)^2}}dx$

$$= \int (\sin x - \cos x)^2 dx$$

$$= \int (1-\sin 2x)dx \quad = x + \frac{\cos 2x}{2} + C$$

13. Integrate : $\int \frac{e^{2x}dx}{1+e^x}.$

Sol^n.—Let $I = \int \frac{e^{2x}}{1+e^x}dx$

Put $e^x + 1 = z$

$e^x dx = dz$

$$\therefore \quad I = \int \frac{e^x \cdot e^x dx}{1+e^x} = \int \frac{(z-1)\cdot dz}{z} \quad = \int\left(1-\frac{1}{z}\right)dz$$

$$= z - \log|z| + C$$
$$= e^x + 1 - \log(e^x+1) + C$$
$$= e^x - \log(e^x+1) + (C+1)$$
$$= e^x - \log(e^x+1) + K$$

14. Evaluate $\int_0^{\pi/2} \sin^4\theta \, d\theta$

Sol^n.—$\int_0^{\frac{\pi}{2}} \sin^4\theta d\theta = \int_0^{\frac{\pi}{2}} (\sin^2\theta)^2 d\theta$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2\theta}{2}\right)^2 d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left[1+\cos^2 2\theta - 2\cos 2\theta\right] d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left[1+\frac{1+\cos 4\theta}{2} - 2\cos 2\theta\right] d\theta$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} \left[2+1+\cos 4\theta - 4\cos 2\theta\right] d\theta$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} \left[3+\cos 4\theta - 4\cos 2\theta\right] d\theta$$

$$= \frac{1}{8}\left[3\theta + \frac{\sin 4\theta}{4} - \frac{4\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{8}\left[3\times\frac{\pi}{2} + \frac{\sin 4\frac{\pi}{2}}{4} - \frac{4\sin 2\frac{\pi}{2}}{2}\right]$$

$$= \frac{1}{8}\left[\frac{3\pi}{2}+0-0\right] = \frac{3\pi}{16}$$

15. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx.$

Sol^n.—Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(i)

then $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(ii)

From (i) + (ii)

$$2I = \int_0^{\frac{\pi}{2}} \left[\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right]dx$$

$$= \int_0^{\frac{\pi}{2}} dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

16. For what values of x, the value of function $y = x^2 - 5x + 3$ is decreasing ?

Sol^n.—Given function $y = x^2 - 5x + 3$ then $\frac{dy}{dx} = 2x - 5$

If the given function is decreasing function then $\frac{dy}{dx} < 0$

$\Rightarrow \quad 2x - 5 < 0$

$\Rightarrow \quad 2x < 5$

$\Rightarrow \quad x < \frac{5}{2}$

17. Solve : $y - x\frac{dy}{dx} = 2(y^2 + \frac{dy}{dx}).$

Sol^n.—Given equation $y - x\frac{dy}{dx} = 2\left(y^2+\frac{dy}{dx}\right)$

$$\Rightarrow \quad y - x\frac{dy}{dx} = 2y^2 + 2\frac{dy}{dx}$$

$$\Rightarrow \quad y - 2y^2 = (x+2)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dx}{x+2} = \frac{dy}{y-2y^2}$$

$$\Rightarrow \quad \frac{dx}{x+2} + \frac{dy}{y(2y-1)} = 0$$

$$\Rightarrow \quad \int\frac{dx}{x+2} + \int\left(\frac{2}{2y-1} - \frac{1}{y}\right)dy = 0$$

$$\Rightarrow \quad \log(x+2) + \log(2y-1) - \log y = \log k$$

$$\Rightarrow \quad \frac{(x+2)(2y-1)}{y} = k$$

18. Solve : $\dfrac{dy}{dx}+2y\tan x=\sin x.$

Soln.—Given differential equation $\dfrac{dy}{dx}+(2\tan x)y=\sin x$ ...(i)

Which is linear differential equation in the form $\dfrac{dy}{dx}+py=Q$

Here, $P = 2\tan x$ and $Q = \sin x$

Now, $I.F.=e^{\int pdx}=e^{2\int \tan x dx}=e^{2\log\sec x}=e^{\log\sec^2 x}$

$\therefore$ $I.F = \sec^2 x$

Hence solution is

$$y(I.F.)=\int Q\cdot(I.F)dx+C$$

or, $y\cdot\sec^2 x=\int \sin x\cdot\sec^2 x dx+C$

$=\int\dfrac{\sin x}{\cos x}\sec x dx+C$

$=\int\tan x\cdot\sec x dx+C=\sec x+C$

$\therefore$ $y=\dfrac{\sec x}{\sec^2 x}+\dfrac{C}{\sec^2 x}=\dfrac{1}{\sec x}+\dfrac{c}{\sec^2 x}$

$y=\cos x+C\cos^2 x$

19. Find the acute angle between the lines whose direction ratios are (1, 1, 2) and $(\sqrt{3}-1,-\sqrt{3}-1,4)$.

Soln.— $\cos\theta=\dfrac{1(\sqrt{3}-1)+1(-\sqrt{3}-1)+2\times4}{\sqrt{1+1+4}\sqrt{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2+4^2}}$

$=\dfrac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}+16}}$

$=\dfrac{6}{\sqrt{6}\sqrt{24}}=\dfrac{6}{\sqrt{6}\times2\sqrt{6}}=\dfrac{6}{6\times2}=\dfrac{1}{2}$

$\Rightarrow$ $\cos\theta=\dfrac{1}{2}=\cos\dfrac{\pi}{3}$ $\Rightarrow$ $\theta=\dfrac{\pi}{3}$

20. Find the value of k so that the lines $\dfrac{x-1}{-1}=\dfrac{y+3}{k}=\dfrac{z-7}{2}$ and $\dfrac{x+1}{k}=\dfrac{y}{2}=\dfrac{z+6}{-3}$ are perpendicular to each other.

Soln.—Given lines $\dfrac{x-1}{-1}=\dfrac{y+3}{k}=\dfrac{z-7}{2}$ ...(i)

and $\dfrac{x+1}{k}=\dfrac{y}{2}=\dfrac{z+6}{-3}$ ...(ii)

If lines (i) and (ii) perpendicular to each other
then $k\times(-1)+2\times k+(-3)\times2=0$

$\Rightarrow$ $-k+2k-6=0$

$\Rightarrow$ $k-6=0$ $\Rightarrow$ $k=6$

21. If $\vec{a}=-4\vec{i}+7\vec{j}-11\vec{k}$ and $\vec{b}=10\vec{i}+\vec{j}+\vec{k}$ then find $\vec{a}\times\vec{b}$.

Soln.—We know that $\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\-4&7&-11\\10&1&1\end{vmatrix}$

$=(7+11)\vec{i}-(-4+110)\vec{j}+(-4-70)\vec{k}$

$=18\vec{i}-106\vec{j}-74\vec{k}$

22. A coin is tossed 4 times. Find the probability that exactly 3 heads appear.

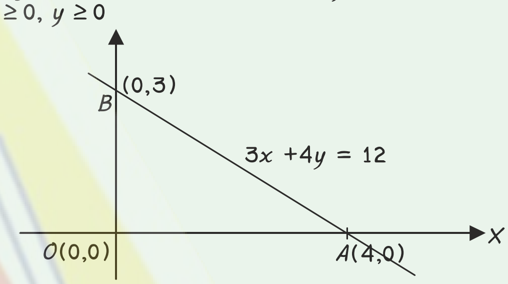
Soln.—Let p = the probability of appearing a head in one trial $=\dfrac{1}{2}$

and q = the probability of appearing a tail in one trial $=\dfrac{1}{2}$

Then the probability of appearing 3 heads in tossing one coin 4 times $={}^4C_3\left(\dfrac{1}{2}\right)^1\left(\dfrac{1}{2}\right)^3={}^4C_3\left(\dfrac{1}{2}\right)^4=\dfrac{\lfloor 4}{\lfloor 3\,\lfloor 1}\times\dfrac{1}{16}=\dfrac{1}{4}$

23. Solve the following linear programming problem :
Maximize $z = 2x + 3y$
subject to the constraints $3x + 4y \le 12$
$x \ge 0,\ y \ge 0.$

Soln.—Given that objective function $z = 2x + 3y$
Subject to the constraints $3x + 4y \le 12$
$x \ge 0,\ y \ge 0$



Now, $3x + 4y = 12$ $\Rightarrow$ $\dfrac{x}{4}+\dfrac{y}{3}=1$

The vertices of feasible region are $O(0,0)$, $A(4,0)$ and $B(0,3)$ then

Corner Point	$z = 2x + 3y$
$O(0,0)$	0 (minimum)
$A(4,0)$	8
$B(0,3)$	9 (maximum)

Hence maximum value of $z = 9$

24. Find the rate of change of the area of a circle with radius r = 8 cm with respect to its radius r.

Soln.—Let area of circle = A
We know that $A = \pi r^2$
Differentiating both sides w.r.t. r

$\dfrac{dA}{dr}=2\pi r$ $\therefore$ $\left.\dfrac{dA}{dr}\right]_{at\ r=8}=2\pi\times8=16\pi cm^2/cm$

25. Evaluate : $(2\vec{i}-3\vec{j}+4\vec{k}).(7\vec{i}+8\vec{j}-9\vec{k})\times(9\vec{i}+5\vec{j}-5\vec{k})$.

Soln.— $\left(7\vec{i}+8\vec{j}-9\vec{k}\right)\times\left(9\vec{i}+5\vec{j}-5\vec{k}\right)=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\7&8&-9\\9&5&-5\end{vmatrix}$

$=(-40+45)\vec{i}-(-35+81)\vec{j}+(35-72)\vec{k}$

$=5\vec{i}-46\vec{j}-37\vec{k}$

$\therefore$ $\left(2\vec{i}-3\vec{j}+4\vec{k}\right)\cdot\left(7\vec{i}+8\vec{j}-9\vec{k}\right)\times\left(9\vec{i}+5\vec{j}-5\vec{k}\right)$

$=\left(2\vec{i}-3\vec{j}+4\vec{k}\right)\cdot\left(5\vec{i}-46\vec{j}-37\vec{k}\right)$

$= 2\times5 + (-3)(-46) + 4\times(-37) = 10 + 138 - 148$

$= 148 - 148 = 0$

26. Is the function $f: N \to N$ onto where $f(x) = 3x, x \in N$?

Soln.—To test whether f is onto

Let $y \in$ co-domain $N \Rightarrow y$ is a natural number

Let $y = f(x) \Rightarrow y = 3x$

$$\Rightarrow x = \frac{y}{3} \notin N \text{ for all } y \in N.$$

Hence $f(x)$ is not onto function.

27. If for two events A and B, $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$ then find $P(A \cup B)$.

Soln.—Given that $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$

We know that $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad 0.4 = \dfrac{P(A \cap B)}{0.8}$

$\Rightarrow \quad 0.4 \times 0.8 = P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.32$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.5 - 0.32$

$= 1.3 - 0.32$

$= 0.98$

$\therefore \quad P(A \cup B) = 0.98$

28. Two dice are thrown simultaneously. Find the probability of getting a sum 9 or 10.

Soln.—Here $n(S) = 36$

Let A = chance of getting (9) = (3,6), (4,5), (5,4), (6,3)

$\therefore \quad n(A) = 4$ then $P(A) = \dfrac{4}{36} = \dfrac{1}{9}$

and Let B = chance of getting (10) = (4,6), (5,5), (6,4)

$\therefore \quad n(B) = 3$ then $P(B) = \dfrac{3}{36} = \dfrac{1}{12}$

$\therefore \quad P(A \cup B) = P(A) + P(B)$

$$= \frac{1}{9} + \frac{1}{12} = \frac{4+3}{36} = \frac{7}{36}$$

29. Evaluate : $\displaystyle\int_a^b \frac{(\log x)^2}{x} dx.$

Soln.—Let $I = \displaystyle\int_a^b \frac{(\log x)^2}{x} dx$

Put $\log x = z \Rightarrow \dfrac{1}{x} dx = dz$

when $x = a$, $z = \log a$

$x = b$, $z = \log b$

$$\therefore \quad I = \int_{\log a}^{\log b} z^2 dz = \left[\frac{z^3}{3}\right]_{\log a}^{\log b} = \frac{(\log b)^3 - (\log a)^3}{3}$$

30. Solve : $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$.

Soln.—Given equation $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$ ...(i)

Putting $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

Now Putting the value of y and $\dfrac{dy}{dx}$ in equation (i)

We get, $v + x\dfrac{dv}{dx} = \dfrac{x - vx}{x + vx}$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-v}{1+v} - v = \frac{1-v-v-v^2}{1+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-2v-v^2}{1+v}$$

$$\Rightarrow \quad \int \frac{1+v}{v^2+2v-1} dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{2(1+v)}{v^2+2v-1} dv + \int \frac{dx}{x} = 0$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{2v+2}{v^2+2v-1} dv + \int \frac{dx}{x} = 0$$

$$\Rightarrow \quad \frac{1}{2}\log(v^2+2v-1) + \log x = \log c$$

$$\Rightarrow \quad \frac{1}{2}\log\left[\frac{y^2}{x^2} + 2\cdot\frac{y}{x} - 1\right] + \log x = \log c$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{y^2+2xy-x^2}{x^2}\right) + \log x = \log c$$

$$\Rightarrow \quad \log\frac{\sqrt{y^2+2xy-x^2}}{x} + \log x = \log c$$

$$\Rightarrow \quad \log\sqrt{y^2+2xy-x^2} - \log x + \log x = \log c$$

$$\Rightarrow \quad \log\sqrt{y^2+2xy-x^2} = \log c$$

$$\Rightarrow \quad \sqrt{y^2+2xy-x^2} = c$$

Long Answer Type Questions

Question Nos. 31 to 38 are Long Answer Type questions. Answer any 4 questions. Each question carries 5 marks. $4 \times 5 = 20$

31. Evaluate : $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$

Soln.—Let $\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & 1+b^2 & bc \\ ac & bc & 1+c^2 \end{vmatrix}$

$[C_1 \to aC_1, C_2 \to bC_2, C_3 \to cC_3]$

$$= \frac{1}{abc}\begin{vmatrix} a(a^2+1) & ab^2 & ac^2 \\ a^2b & b(b^2+1) & bc^2 \\ a^2c & b^2c & c(c^2+1) \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

By $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

By, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= (1 + a^2 + b^2 + c^2)(1 - 0) = 1 + a^2 + b^2 + c^2$

32. Prove that, $\tan^{-1}\frac{2}{5}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{12}=\frac{\pi}{4}$.

Solⁿ.—L.H.S. $=\tan^{-1}\left(\frac{2}{5}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{12}\right)$

$$=\tan^{-1}\left(\frac{\frac{2}{5}+\frac{1}{3}}{1-\frac{2}{5}\times\frac{1}{3}}\right)+\tan^{-1}\left(\frac{1}{12}\right)$$

$$=\tan^{-1}\left(\frac{11}{13}\right)+\tan^{-1}\left(\frac{1}{12}\right)$$

$$=\tan^{-1}\left(\frac{\frac{11}{13}+\frac{1}{12}}{1-\frac{11}{13}\times\frac{1}{12}}\right)$$

$$=\tan^{-1}\left(\frac{132+13}{156-11}\right)=\tan^{-1}\left(\frac{145}{145}\right)$$

$$=\tan^{-1}(1)=\frac{\pi}{4}=\text{R.H.S.}$$

33. If $(x+y)=\sec^{-1}(x-y)$ then find $\frac{dy}{dx}$.

Solⁿ.—Given that $x+y=\sec^{-1}(x-y)$

$\Rightarrow\ \sec(x+y)=(x-y)$

Differentiating both sides w.r.t x

$$\sec(x+y)\tan(x+y)\left[1+\frac{dy}{dx}\right]=1-\frac{dy}{dx}$$

or, $$\left[\sec(x+y)\tan(x+y)+1\right]\frac{dy}{dx}=1-\sec(x+y)\tan(x+y)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{1-\sec(x+y)\tan(x+y)}{1+\sec(x+y)\tan(x+y)}$$

34. Minimize $z=2x+y$

$x+y\geq 1$

$x+2y\leq 10$

$y\leq 4$

$x\geq 0,\ y\geq 0$

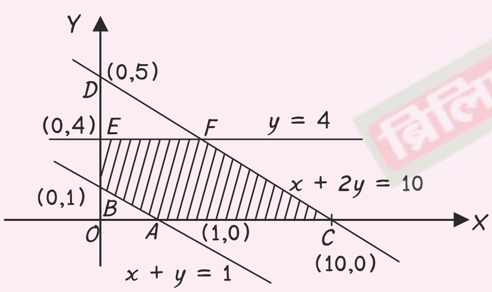
Solⁿ.—Since $x\geq 0$, $y\geq 0$ therefore the relevent region will be in the first quadrant.

The equation corresponding to the constraints

$x+y=1$

$x+2y=10$

and $y=4$



Here feasible region is ACFEBA which is shaded.

Corner points of feasible region are $A(1,0)$, $C(10,0)$, F (2,4), E (0,4), B (0,1)

Corner Point	$z=2x+y$
(1,0)	2
(10,0)	20
(2,4)	8
(0,4)	4
(0,1)	1

$\therefore$ The value of z is minimum at (0,1)

and minimum value of $z=1$

Hence for minimum $z=1$ for $x=0$, $y=1$

35. Solve : $x\cos\left(\frac{y}{x}\right)(y\,dx+x\,dy)=y\,(x\,dy-y\,dx)\sin\left(\frac{y}{x}\right)$.

Solⁿ.—Given equation $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y(xdy-ydx)\sin\left(\frac{y}{x}\right)$

$$\Rightarrow\ \frac{x}{y}(ydx+xdy)=(xdy-ydx)\frac{\sin\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right)}$$

$$\Rightarrow\ \frac{x}{y}\frac{(ydx+xdy)}{x^2}=\frac{(xdy-ydx)}{x^2}\tan\left(\frac{y}{x}\right)$$

or, $$\frac{ydx+xdy}{xy}=\tan\left(\frac{y}{x}\right)\left(\frac{xdy-ydx}{x^2}\right)$$

or, $$\int\frac{d(xy)}{xy}=\int\tan\left(\frac{y}{x}\right)d\left(\frac{y}{x}\right)$$

$$\Rightarrow\ \log(xy)=\log\sec\left(\frac{y}{x}\right)+\log k$$

$$\Rightarrow\ \log(xy)-\log\sec\left(\frac{y}{x}\right)=\log k$$

$$\Rightarrow\ \log\left[\frac{xy}{\sec\left(\frac{y}{x}\right)}\right]=\log k$$

$$\Rightarrow\ \frac{xy}{\sec\left(\frac{y}{x}\right)}=k$$

$$\Rightarrow\ xy=k\sec\left(\frac{y}{x}\right)$$

$$\Rightarrow\ xy\cos\left(\frac{y}{x}\right)=k$$

36. Find the shortest distance between the lines

$\vec{r}=(\vec{i}+2\vec{j}+\vec{k})+\lambda(\vec{i}-\vec{j}+\vec{k})$ and

$\vec{r}=(2\vec{i}-\vec{j}-\vec{k})+\mu(2\vec{i}+\vec{j}+2\vec{k})$

Solⁿ.—Given lines $\vec{r}=(\vec{i}+2\vec{j}+\vec{k})+\lambda(\vec{i}-\vec{j}+\vec{k})$

and $\vec{r}=(2\vec{i}-\vec{j}-\vec{k})+\mu(2\vec{i}+\vec{j}+2\vec{k})$

Let $\vec{a}_1 = \vec{i} + 2\vec{j} + \vec{k}, \vec{b}_1 = \vec{i} - \vec{j} + \vec{k}$

and $\vec{a}_2 = 2\vec{i} - \vec{j} - \vec{k}, \vec{b}_2 = 2\vec{i} + \vec{j} + 2\vec{k}$

then $\vec{a}_2 - \vec{a}_1 = \vec{i} - 3\vec{j} - 2\vec{k}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$$= (-2-1)\vec{i} - (2-2)\vec{j} + (1+2)\vec{k} = -3\vec{i} + 3\vec{k}$$

The required shortest distance

$$= \frac{\left|\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right)\right|}{\left|\vec{b}_1 \times \vec{b}_2\right|}$$

$$= \frac{\left|\left(\vec{i} - 3\vec{j} - 2\vec{k}\right)\cdot\left(-3\vec{i} + 3\vec{k}\right)\right|}{\left|-3\vec{i} + 3\vec{k}\right|}$$

$$= \frac{|-3-6|}{\sqrt{9+9}} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ units}$$

37. Find the mean of the number of tails in the throw of three coins.

Solⁿ.—Let S be the sample space of tossing three coins

then $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore \quad n(S) = 8$

Let X denote the number of tails occured, then

$P(X = 0)$ = probability of occurrence of zero tail $= P(HHH)$

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

$P(X = 1)$= Probability of occurrence of one tail = P(HHT) + P(HTH) + P(THH)

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$P(X = 2)$ = Probability of occurrence of two tails = $P(HTT) + P(THT) + P(TTH)$

$$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{3}{8}$$

$P(X = 3)$ = Probability of occurrence of three tails

$$= P(TTT) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

Thus the probability distribution when three coins are tossed is a given below

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Hence Mean $= \mu = \Sigma p_i x_i$

$$= \frac{1}{8}\times 0 + \frac{3}{8}\times 1 + \frac{3}{8}\times 2 + \frac{1}{8}\times 3$$

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{3+6+3}{8}$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5$$

38. Evaluate : $\int_0^{\pi/2} \log \sin x \, dx$.

Solⁿ.—Let $I = \int_0^{\frac{\pi}{2}} \log \sin x dx$ ...(i)

then $I = \int_0^{\pi} \log \sin\left(\frac{\pi}{2} - x\right)dt$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log \cos x dx$ ...(ii)

From (i) + (ii)

$$2I = \int_0^{\frac{\pi}{2}} [\log \sin x + \log \cos x]\, dx$$

$$= \int_0^{\frac{\pi}{2}} \log \sin x \cos x dx$$

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{2\sin x \cos x}{2}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log \frac{\sin 2x}{2} dx$$

$$= \int_0^{\frac{\pi}{2}} [\log \sin 2x - \log 2]\, dx$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \log \sin 2x dx - \log 2 \int_0^{\frac{\pi}{2}} dx$$

$$\Rightarrow \quad 2I = I_1 - \log 2\,[x]_0^{\frac{\pi}{2}}$$

where $\quad I_1 = \int_0^{\frac{\pi}{2}} \log \sin 2x dx$

$\Rightarrow \quad 2I = I_1 - \frac{\pi}{2}\log 2$ ...(iii)

For $I_1 = \int_0^{\frac{\pi}{2}} \log \sin 2x dx$

Put $2x = z$

$2dx = dz$

When $x = 0, z = 0$

$x = \frac{\pi}{2}, z = \pi$

$$\therefore \quad I_1 = \int_0^{\pi} \log \sin z \frac{dz}{2} = \frac{1}{2}\int_0^{\pi} \log \sin z dz = \frac{1}{2}\cdot 2\int_0^{\frac{\pi}{2}} \log \sin z dz$$

$$I_1 = \int_0^{\frac{\pi}{2}} \log \sin x dx = I$$

From (iii)

$$2I = I - \frac{\pi}{2}\log 2$$

$$\therefore \quad I = -\frac{\pi}{2}\log 2$$